



# High-precision Harmonic Detection in Convolution

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**(Abstract)** The Fast Fourier transform (FFT) exist frequency domain spectral leakage, picket fence effect, as well as non-integer wave phenomenon. In this paper, a time-frequency filter is designed, which can detect the frequency, amplitude and phase of any order harmonics and interharmonics in signal by means of time domain convolution. Negative half-axis frequency is restrained by Hilbert transform. The theory analysis are carried to this method and the calculate formula are concluded. Experiment simulation results show that time-frequency filtering convolution function can be designed and realized neatly and be real-time implemented conveniently in engineering application; the influences of fundamental frequency fluctuation on harmonic analysis are restrained by using the approach presented in this paper; the calculating frequencies with many order harmonics and interharmonics are nicely, the amplitudes and the initial phases are detected in high-accurately.

**Keywords:** Harmonic Analysis; Time-frequency Filtering; Convolution; Hilbert Transform; Frequency Fluctuation.

## 1. INTRODUCTION

High-precision analysis of harmonic power measurement, harmonic power flow calculation, network testing equipment, power system harmonics compensation and suppression is of great significance[1]. Because non-synchronous sampling and data truncation, the use of fast Fourier transform (FFT) algorithm to generate harmonic analysis and fence effect of spectrum leakage, the accuracy of harmonic analysis[2-3].

In order to eliminate these phenomena, scholars Design a lot of harmonic detection methods based on Fourier transform of many kinds window, such as rectangular [4], Hanning [5], Hamming [6], Blackman [7], Blackman-Harris [8], Kaiser window[9] and various improvements [10-11] and other windowed. Fast Fourier Transform(FFT) can reduce the encounter alone and fence effect of spectrum leakage problems and improve the detection accuracy of the harmonic parameters, but can not detect integer asked near the harmonic harmonic; cosine combination of high-end window-based dual-line[5,7,12] or line[13-14] interpolation FFT algorithm to estimate fundamental and the harmonic parameters, need to solve high-order equation[15-17], the computational complexity; continuous wavelet transform[18-19] can be realized between / harmonic detection, but the wavelet functions of different scales exist in the frequency domain interference, when the detection signal the harmonic components with frequencies close to, the detection method will fail; Prony method[20-21] is harmonic, harmonic analysis and modeling inter-effective way to accurately estimate the sinusoidal component of frequency, amplitude and phase angle , but the need to solve two equations and a

polynomial of odd, high computational complexity and sensitivity to noise; there are other methods[22-24], or limited frequency resolution, or calculate volume, both in the specific application limitations.

This paper presents a time-frequency filter, axle with the Hilbert transform in the frequency of negative, high-precision time domain convolution signal is detected between all the harmonics and the harmonic frequency, amplitude and phase. In this paper, a theoretical analysis and calculation formula is derived, the method to avoid the Fourier (FFT) domain spectral leakage, the entire sub-barrier effect and non-wave phenomenon. The simulation results show that: time-frequency convolution filter design flexible, easy to use, this algorithm can eliminate the harmonic interference and improve signal analysis precision, high accuracy for harmonic analysis.

## II. time-frequency filter design and analysis of Hilbert transform

### A. Time-frequency filter design

Time-frequency filter:

$$g(t, \omega_0) = \sin\left(\frac{a\pi t}{2}\right) e^{j\omega_0 t} (\varepsilon(t) - \varepsilon(t - \frac{2}{a})) \quad (1)$$

Where  $\varepsilon(t)$  is the step function,  $a = 2\pi B$  as the filter center parameters, the coefficient B is used to adjust the filter bandwidth (such as taking  $B = 1$ ),  $\omega_0$  center frequency. The frequency domain expression is:

$$G(\omega, \omega_0) = H(\omega - \omega_0) = \frac{1}{a} [Sa\left(\frac{\omega - \omega_0}{a} + \frac{\pi}{2}\right) + Sa\left(\frac{\omega - \omega_0}{a} - \frac{\pi}{2}\right)]e^{-j\frac{\omega - \omega_0}{a}} \quad (2)$$

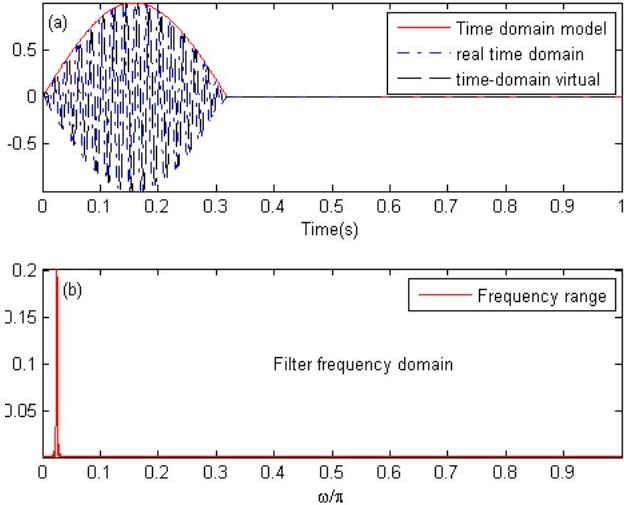


Figure 1 Time-frequency filter of time / frequency characteristics

Where  $Sa(t) = \frac{\sin(t)}{t}$ ,  $Sa(0) = 1$ ; Figure 1 shows the trend of time-frequency filter characteristics, (a) trends in the time domain graph, (b) trends in the frequency domain; they change with the center frequency  $\omega_0$ . By (2) and Figure 1 (b) shows  $G(\omega, \omega_0)$  only in a narrow band centered  $\omega_0$  significant amplitude, the other is almost zero.

## B.Hilbert transform

HT is defined as the real signal [25] :

$$\tilde{f}(t) = f(t) \otimes \frac{1}{\pi t} = \frac{1}{\pi} \int_{-\infty}^{+\infty} \frac{f(\tau)}{t - \tau} d\tau \quad (3)$$

Which  $\otimes$  for the sake of convolution, the Fourier transform  $F(\omega)$ ,  $\tilde{f}(t)$  the Fourier transform

$$\tilde{F}(\omega) = -jSgn(\omega)F(\omega) \quad (4)$$

$Sgn(\omega)$  the sign function. Signal  $f(t)$  through the Hilbert transform and its complex signal link, it can be analytic signal

$$x(t) = f(t) + j\tilde{f}(t) \quad (5)$$

An important property of analytic signal is the positive frequency components retained, excluding the negative frequency part. That

$$X(\omega) = (1 + Sgn(\omega))F(\omega) = \begin{cases} 2F(\omega) & \omega > 0 \\ F(\omega) & \omega = 0 \\ 0 & \omega < 0 \end{cases} \quad (6)$$

## III Theoretical analysis and calculation formulas

### A.Analysis of Continuous

If  $\omega_0$  centered within the range of narrow-band frequency  $\omega_1 > 0$  of the harmonic signal:

$$f(t) = A \cos(\omega_1 t + \phi) \quad (7)$$

Its frequency domain expression:

$$F(\omega) = A\pi[\delta(\omega - \omega_1)e^{j\phi} + \delta(\omega + \omega_1)e^{-j\phi}] \quad (8)$$

The analytic signal is the Fourier transform (from (6), (8), we have)

$$X(\omega) = 2A\pi\delta(\omega - \omega_1)e^{j\phi} \quad (9)$$

$$\begin{aligned} Y(\omega) &= G(\omega, \omega_0)X(\omega) = H(\omega - \omega_0)X(\omega) \\ &= 2A\pi H(\omega_1 - \omega_0)e^{j\phi}\delta(\omega - \omega_1) \end{aligned} \quad (10)$$

$$y(t) = x(t) \otimes g(t, \omega_0) = AH(\omega_1 - \omega_0)e^{j\phi}e^{j\omega_1 t} \quad (11)$$

then:

$$\omega_1 = \frac{d(Arg(y(t)))}{dt} \quad (12)$$

,Arg is Angular

$$A = \frac{|y(t)|}{|H(\omega_1 - \omega_0)|} \quad (13)$$

$$\phi = Arg(y(t)) - mod[\omega_1 t, 2\pi] - Arg(H(\omega_1 - \omega_0)) \quad (14)$$

The remainder mod[.] is divisible .

### B.Calculation of Discrete

Set of discrete sampling frequency  $fs$ , the sampling period  $DT = \frac{1}{fs}$ , Number of samples is  $N$ ; take  $N_1 = [0.5N]$ ,  $N_2 = [0.94N]$ , Computing discrete convolution:

$$y(k) = DT \sum_{i=\max\{1,k-N\}}^{\min\{k-1,N\}} g(i)x(k-i) \quad k = 1, 2, \dots, N \quad (15)$$

$$\begin{aligned} \theta(k) &= Arg(y(k)) - Arg(y(k-1)) \\ &+ 2\pi \max\{0, Sgn(-Arg(y(k)) + Arg(y(k-1)))\} \end{aligned} \quad (16)$$

$k = N_1, \dots, N_2$ ;  $Sgn$  is sign function

The harmonic frequency  $f$  (Hz), amplitude  $A$ , the initial phase  $\phi$  ( $^\circ$ ) as, respectively:

$$f = \frac{fs}{2\pi(N_2 - N_1 + 1)} \sum_{k=N_1}^{N_2} \theta(k) \quad (17)$$

$$\omega_1 = \frac{2\pi f}{fs} \quad (18)$$

$$A = \frac{1}{|H(\omega_1 - \omega_0)| (N_2 - N_1 + 1)} \sum_{k=N_1}^{N_2} |y(k)| \quad (19)$$

$$\psi = \left( \sum_{k=N_1}^{N_2} \{ \text{Arg}(y(k)) + 2\pi \max\{0, \text{Sgn}(-\text{Arg}(y(k))) - \text{mod}[\frac{2\pi(k-1)f}{fs}, 2\pi]\} \} \right) \quad (20)$$

$$\phi = \frac{\frac{1}{(N_2 - N_1 + 1)} - \text{Arg}(H(\omega_1 - \omega_0))}{\pi} \quad (21)$$

### C. Experimental Evaluation

Signal contains fundamental, DC, between 2 and 3 harmonic harmonic, and their parameters in Table 1, the expression:

$$f(t) = \sum_{i=0}^6 A_i \cos(\omega_i t + \phi_i) \quad (22)$$

Its sampling frequency  $fs = 2\text{kHz}$ , the number of samples  $N = 5000$ . This method results are in Table 1 the right department. To the harmonic frequency, amplitude, initial phase of testing the value of the real value and are plotted in the same plot, the result is very accurate.

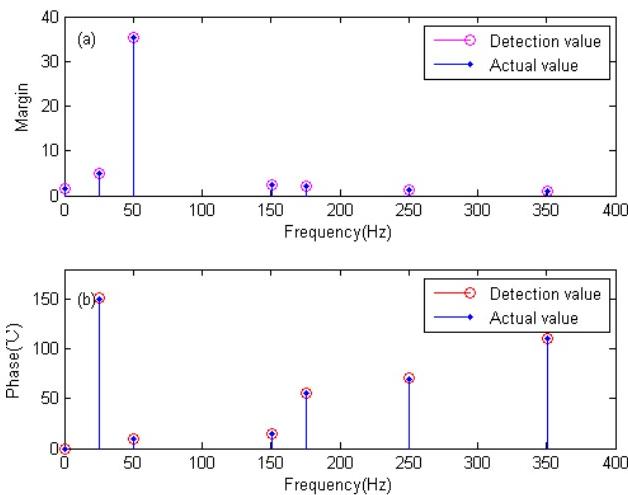


Figure 2 The harmonic characteristics of the true value of parameters compared with the measured values

Table 1 harmonic of the frequency  $f = 350.7\text{Hz}$  specific icon near the detection algorithms 3, Figure (a), (b), (c) of the abscissa as the sample points. Figure (a) to (15) the magnitude, Figure (c) signal after filtering in frequency domain inverse Fourier transform (IFFT) of the amplitude, both in the same 500 points; Figure (b) the type (16) transient frequency, Figure (d) the abscissa is the frequency (Hz), Hilbert transform signal was analytical signal in the frequency domain amplitude

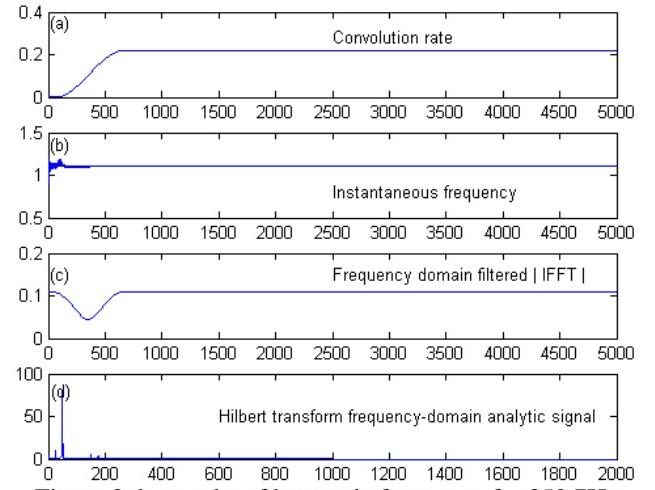


Figure 3 the results of harmonic frequency  $f = 350.7\text{Hz}$

## IV. CONCLUSIONS

This new vision through the time-domain convolution can accurately detect the signal between all the harmonics and the harmonic frequency, amplitude and phase. In this paper, a time-frequency filter is designed with the Hilbert transform in the frequency of negative axle. The theoretical design analysis and calculation formula is derived, the method can be to avoid the Fourier (FFT) domain spectral leakage, barrier effect and non-integer secondary wave phenomenon. The simulation results show that: time-frequency convolution filter design is flexible, project implementation is easy and the algorithm is simple, response is quick. This algorithm can eliminate the harmonic interference and improve signal analysis precision, which can been applied in high accuracy for harmonic analysis. Applications of time-frequency domain filter selection bandwidth should be non-zero values within the used data segment, not to destroy its integrity

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